Explain MinMax algorithm with example

**Minimax** (sometimes **MinMax** or **MM**[[1]](https://en.wikipedia.org/wiki/Minimax#cite_note-1)) is a decision rule used in [decision theory](https://en.wikipedia.org/wiki/Decision_theory), [game theory](https://en.wikipedia.org/wiki/Game_theory), [statistics](https://en.wikipedia.org/wiki/Statistics) and [philosophy](https://en.wikipedia.org/wiki/Philosophy) for *mini*mizing the possible [loss](https://en.wikipedia.org/wiki/Loss_function) for a worst case (*max*imum loss) scenario. Originally formulated for two-player [zero-sum](https://en.wikipedia.org/wiki/Zero-sum) [game theory](https://en.wikipedia.org/wiki/Game_theory), covering both the cases where players take alternate moves and those where they make simultaneous moves, it has also been extended to more complex games and to general decision-making in the presence of uncertainty.

The **maximin value** of a player is the largest value that the player can be sure to get without knowing the actions of the other players.

Calculating the maximin value of a player is done in a worst-case approach: for each possible action of the player, we check all possible actions of the other players and determine the worst possible combination of actions - the one that gives player {\displaystyle i} the smallest value. Then, we determine which action player {\displaystyle i} can take in order to make sure that this smallest value is the largest possible.

For example, consider the following game for two players, where the first player ("row player") may choose any of three moves, labelled T, M, or B, and the second player ("column" player) may choose either of two moves, L or R. The result of the combination of both moves is expressed in a payoff table:

|  |  |  |
| --- | --- | --- |
|  | **L** | **R** |
| T | 3,1 | 2,-20 |
| M | 5,0 | -10,1 |
| B | -100,2 | 4,4 |

(where the first number in each cell is the pay-out of the row player and the second number is the pay-out of the column player).

For the sake of example, we consider only pure strategies. Check each player in turn:

* The row player can play T, which guarantees him a payoff of at least 2 (playing B is risky since it can lead to payoff -100, and playing M can result in a payoff of -10). Hence: {\displaystyle {\underline {v\_{row}}}=2}.
* The column player can play L and secure a payoff of at least 0 (playing R puts him in the risk of getting -20). Hence: {\displaystyle {\underline {v\_{col}}}=0}.

If both players play their maximin strategies (T,L), the payoff vector is (3,1). In contrast, the only [Nash equilibrium](https://en.wikipedia.org/wiki/Nash_equilibrium) in this game is (B,R), which leads to a payoff vector of (4,4).

The **minimax value** of a player is the smallest value that the other players can force the player to receive, without knowing his actions. Equivalently, it is the largest value the player can be sure to get when he *knows* the actions of the other players. Its formal definition is:[[2]](https://en.wikipedia.org/wiki/Minimax" \l "cite_note-ZMS2013-2)

{\displaystyle {\overline {v\_{i}}}=\min \_{a\_{-i}}\max \_{a\_{i}}{v\_{i}(a\_{i},a\_{-i})}}The definition is very similar to that of the maximin value - only the order of the maximum and minimum operators is inverse.

For every player *i*, the maximin is at most the minimax:

{\displaystyle {\underline {v\_{i}}}\leq {\overline {v\_{i}}}}

Intuitively, in maximin the maximization comes before the minimization, so player *i* tries to maximize their value before knowing what the others will do; in minimax the maximization comes after the minimization, so player *i* is in a much better position - they maximize their value knowing what the others did.

Usually, the maximin is strictly smaller than the minimax. Consider the game in the above example:

* The row player can get a value of 4 (if the other player plays R) or 5 (if the other player plays L), so: {\displaystyle {\overline {v\_{row}}}=4}.
* The column player can get 1 (if the other player plays T), 1 (if M) or 4 (if B). Hence: {\displaystyle {\overline {v\_{col}}}=1}.

**Example**[[edit](https://en.wikipedia.org/w/index.php?title=Minimax&action=edit&section=4" \o "Edit section: Example)]

|  |  |  |  |
| --- | --- | --- | --- |
|  | **B chooses B1** | **B chooses B2** | **B chooses B3** |
| **A chooses A1** | +3 | −2 | +2 |
| **A chooses A2** | −1 | 0 | +4 |
| **A chooses A3** | −4 | −3 | +1 |

The following example of a zero-sum game, where **A** and **B** make simultaneous moves, illustrates *minimax*solutions. Suppose each player has three choices and consider the [payoff matrix](https://en.wikipedia.org/wiki/Payoff_matrix) for **A** displayed on the right. Assume the payoff matrix for **B** is the same matrix with the signs reversed (i.e. if the choices are A1 and B1 then **B** pays 3 to **A**). Then, the minimax choice for **A** is A2 since the worst possible result is then having to pay 1, while the simple minimax choice for **B** is B2 since the worst possible result is then no payment. However, this solution is not stable, since if **B** believes **A** will choose A2 then **B** will choose B1 to gain 1; then if **A** believes **B** will choose B1 then **A** will choose A1 to gain 3; and then **B** will choose B2; and eventually both players will realize the difficulty of making a choice. So a more stable strategy is needed.

Some choices are *dominated* by others and can be eliminated: **A** will not choose A3 since either A1 or A2 will produce a better result, no matter what **B** chooses; **B** will not choose B3 since some mixtures of B1 and B2 will produce a better result, no matter what **A** chooses.

**A** can avoid having to make an expected payment of more than 1∕3 by choosing A1 with probability 1∕6 and A2 with probability 5∕6: The expected payoff for **A** would be 3 × (1∕6) − 1 × (5∕6) = −1∕3 in case **B** chose B1 and −2 × (1∕6) + 0 × (5∕6) = −1/3 in case **B** chose B2. Similarly, **B** can ensure an expected gain of at least 1/3, no matter what **A** chooses, by using a randomized strategy of choosing B1 with probability 1∕3 and B2 with probability 2∕3. These [mixed](https://en.wikipedia.org/wiki/Mixed_strategy) minimax strategies are now stable and cannot be improved.